PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

Volume 5 MARCH 15, 1919 Number 3

NOTE ON THE SELF-ADJUSTING INTERFEROMETER IN RELATION TO THE ACHROMATIC FRINGES

By Carl Barus

Department of Physics, Brown University

Communicated, January 3, 1919

1. Introductory.—This apparatus was first used for the case of coincident ray systems by Michelson and Morley[1] in their famous experiments on the Fresnel-Fizeau coefficient and has since been similarly employed by Zeeman.[2] It has so many practical advantages that a special reference here is justified. It is shown in a modified form[3] in figure 1, where L is a pencil of white light preferably from a collimator. It is separated by the half silvered plate N into the two beams $L12345T$ and $L16785T$, both of which are recombined at 5 and then enter the telescope at T. It is merely necessary to rotate any of the mirrors, say N, around a vertical axis until the two vertical white wide slit images coincide in the telescope, when brilliant fringes will be at once obtained on the coincident white fields. The central fringe is achromatic, for the system is self-compensating, or the glass paths are rigorously equal. The fringes may be enlarged to infinite size and then reduced in size again, the phenomenon being symmetrical, by rotating any mirror, say N, about a horizontal axis.

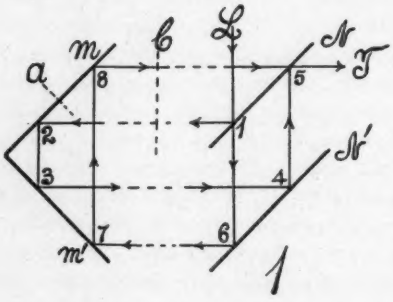
The mirrors N and N' are rigidly fixed to a carriage capable of sliding right and left parallel to the lines 85, etc. Hence the rays 85 and 21 and the rays 34 and 76 may be brought to coincidence or separated in any degree, at pleasure. If the slide were perfect the fringes would not be disturbed by this process; but few slides are perfect to this degree, and the fringes will change size somewhat, since there is rotation. Practically this is of no consequence.

The fringes, which when sharp are necessarily horizontal, may also be changed in size by inserting a plate glass compensator about 5 mm. thick in the paired beams 85, 21, or 34, 76. When this is rotated on a horizontal axis, the fringes pass through infinite size, and this arrangement is particularly adapted for the detection of the character of the fringes and will be so used presently.

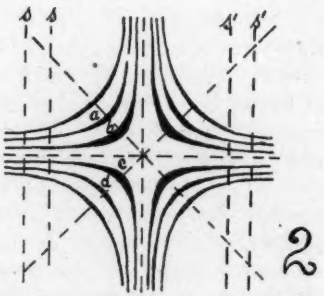
If a direct vision prism or grating is placed in front of the telescope, the spectrum is seen to be crossed by intense black lines, very nearly parallel and horizontal, but actually diverging from blue to red symmetrically up and down from the horizontal central black line. It is not necessary here, that the slit be fine. In fact it may be several millimeters broad without destroying these spectrum fringes, if essentially horizontal.

If the linear phenomenon of reversed spectra, coinciding on a certain line of the spectrum is wanted, a prism may be inserted between m and m' suitably adjusted. These fringes then also appear at once and may be put in any color, at pleasure, on rotating any mirror, say N, on a vertical axis. Rotation on a horizontal axis enlarges.

Finally if separate plate glass compensators are placed in the paired beams 85 and 21, for instance, and rotated around a horizontal axis independently, the fringes may be displaced up or down the slit image for the purpose of measurement. A double offset air compensator consisting of 3 right-angled



1



2

V-mirrors with their corresponding faces parallel, the central V-mirror movable on a micrometer (described in my paper on gravitation) is particularly available. Such a compensator would be placed normally to the rays 85 and 21, for instance, to give them lateral path length. In these cases the spectrotelescope may also be used, where the strong bands register the displacement in any wave length. Since the slit may be broad, there is here also a great abundance of light.

The rays 85, 21, may be made of almost any reasonable length, and distance apart, if the mirrors N, N', m, m', are broad. To secure greater length, the mirrors mm' (rigidly connected) may be moved at pleasure in the direction 58, without disturbing the fringes, good slides presupposed. The rays 85 and 21, 34 and 76 may be separated, if x is the available breadth of mirror to an extent $x \cos 45°$, by moving the rigid system NN' in the direction 85.

If either of the mirrors m or m' is separated (as for instance at a in figure 6), the part may be placed on a micrometer; but the apparatus would not then be selfadjusting; for the parts will not in general be coplanar.

To secure the best conditions for sharp strong fringes the two slit images seen in the telescope must be of equal intensity, and this depends on the half-silver N. On testing a number of plates it is usually easy to find one which fulfills this condition nearly enough. The fringes are still good, even when the intensity of images is noticeably different. The secondary images due to the reflection from glass faces are either weak or (if thick plates are used) these reflections may be blotted out by small opaque screens suitably fixed to the mirrors m and N. It is interesting to observe that with proper adjustment these secondary reflections carry their own fringes, some of which are modified in like conditions more rapidly than the main set. Intersecting fringes producing a beaded structure and fringes moving in opposite directions are also observed.

2. Character of the achromatic fringes.—Since the achromatic fringes are quite symmetric, consisting of a central white or fringe, flanked on either side by 3 or 4 colored fringes rapidly decreasing in intensity, it is obvious that (practically) they must consist of superposed monochromatic confocal hyperbolae. This may be well shown in the present apparatus, where the fringes are stationary and are displayed relative to horizontal and vertical lines of symmetry. To carry out the experiment, it is best to insert a single plate compensator (say C, fig. 6, 5 mm. thick) normally into the rays 85 and 21, preferably in the same vertical plane. When the plate is not perfect, it may be necessary to adjust for coincidence of slit images. If now this plate is rotated about a horizontal axis (normal to the lines 85, 21) the fringes walk laterally through the broad coincident slit images, in such a way as to clearly outline a moving design of the form given in figure 2. In other words, as a first approximation (for the case is, of course, essentially more complicated) the achromatic fringes may be assumed to be a family of confocal equilateral hyperbolas, referred to given horizontal and vertical axes. When the rays 85, 12 are at the same level, the broad slit image is in a position of symmetry relating to the hyperbolae, figure 2. When this is not the case, the image is at ss or $s's'$, with the fringes very rapidly becoming horizontal. Since this design is similarly carried out with decreasing coarseness from red to violet, it is clear that a single characteristic central achromatic fringe results, invaluable for purposes of displacement interferometry, from its smallness and since from the breadth of slit it can be made so intensely luminous. When the path difference of the rays 85, 12, in figure 1 is changed by the micrometer or by independent compensators, the figure 2 shifts bodily up or down the slit image. It is also obvious that when the fringes with white light are horizontal, they must appear as horizontal black bands in the spectro-telescope, regardless of the width of slit used; and hence these fringes also are excessively luminous, while their displacement may be referred to a definite wave length. If the interferometer is not selfadjusting, the axes of figure 2 are as a rule inclined, and fringes are obtained in all angles of altitude needing special adjustment. The spectrum fringes then demand a fine slit but are also hori-

zontal. The shift of achromatic fringes due to micrometer displacement may therefore be at once expressed in terms of the spectrum fringes rigorously in a given wave length.

A very interesting transformation of the design, figure 2, will be noticed, if by rotating any of the mirrors, N, m, figure 1 for instance, the two white slit images seen in the telescope, are passed horizontally through each other. During this motion the originally vertical, nearly linear achromatic fringe, passes through the form of the area between the hyperbolae a and b, figure 2; next through the area between b and c (coincidence of slit images); then into the area between c and d; finally again into a vertical hair line, always retaining its individuality among the surrounding colored fringes of similar shapes. The whole is particularly vivid if the fringes are observed with the ocular drawn well forward, quite out of focus. The same may be done by adding a diopter spectacle lens (convex or concave), to the objective. The bearing of this will be seen presently.

3. Curved compensators.—If the rays 85, 21 in figure 1 are brought to coincidence, it is obvious that a lens, either convex or concave, may be inserted between the mirrors m and m' and normal to the rays, without destroying the interferences, though they must be greatly modified in form. If the lens is symmetrically inserted, the two broad slit images will be equally wide, so that coincidence is perfect. The fringes so obtained are usually large, brilliantly colored circles. In case of imperfect plate they become oval and coarse. The large central disc is achromatic. To center the fringes the mirror m' may be rotated on a vertical and horizontal axis, until the symmetrical circular figure is obtained. Here again the individuality and even the approximate position of the achromatic fringe is retained on passing the broad slit images through each other; but the sequences of types of fringe is peculiar. As the slit images separate toward the right or the left, because of the corresponding rotation of m on a vertical axis, the originally colorless disc of the circular fringes not only moves to the right or left, but at the same time becomes very vividly colored. The coarse fringes now show considerable resemblance to the coronas of cloudy condensation, in which there is also a colored disc. When the slit images have been markedly separated, the disc vanishes and thinner lines appear, at first as complete circles surrounding the fading disc, but rapidly losing curvature to become vertical. Throughout the whole transformation there has been a grouping of symmetrically concentric colored circles, on both sides of the achromatic circle. To state this differently: each originally linear fringe, in turn, on expanding (slit images approaching coincidence), contracts vertically and broadens horizontally into a disc, which retains the color of the fringe out of which it originates. The same result may be obtained by moving the lens inserted between m and m' into both rays, fore and aft (direction 8, 5). Similarly the corresponding sequence between horizontal fringes appears, on moving the lens up and down. If the m, m', mirrors are moved bodily fore and aft, however, the circular

fringes merely pass horizontally through the field without appreciably changing form.

It makes little difference whether concave or convex lenses are inserted between m and m', except that the objective of the telescope will have to be armed with a convex or a concave lens (of the same strength) respectively, to assist in focussing. But here again the most vivid effects are obtained with the ocular drawn out of focus. I examined lenses of 1, 2, and 3 diopters of focal power, concave and convex. There would be nothing against the treatment of stronger lenses, only the secondary adjustments become increasingly difficult, unless special devices are resorted to. Many of the forms are quite visible to the naked eye.

If the lens is not symmetrical in form, i.e., for plano-convex meniscus and other lenses, the simple figures above discussed become more complicated fand the fringes multi-annular.

4. Index of refraction, irrespective of form.—If a plane-parallel trough containing a solution of mercury potassic iodide is placed between m and m', figure 1, normally to the rays, neither the achromatics nor the spectrum fringes (broad slit admissible) are affected. Inclination to the normal position will change the size of fringes only. Hence if a piece of glass is inserted into the trough with the rays separated as at 2, 3 and 8, 7 in figure 1, one of them (say 8, 7) only passing through the glass, the spectrum fringes will change form or vanish except at that part of the spectrum in which the index of refraction of the glass and of the solution are identical supposing the dispersion coefficients to be nearly enough the same. It is thus of interest to determine to what degree this method can be practically utilized.

In the case of bodies of regular form, like lenses, spectrum fringes and even achromatics will usually appear, when the sharply seen, fine slit images coincide in the principal focus (i.e., the position of the ocular for parallel rays). But the fringes will as a rule be in other focal planes.

5. The same. Glass plate.—In relation to the principle, §4, in question, if a plate of glass of higher refractive index is introduced into the solution and traversed by one ray only, the original intensely black, nearly horizontal bands in the spectrum are changed to much finer lines, at a considerable angle (45°, etc.) to the horizontal. This inclination is symmetrically down toward the red, or up toward the red, according as one beam or the other traverses the glass. Moreover the size of the fringes now decreases, in much more pronounced ratio from red to violet. The achromatics have necessarily vanished. It would need special compensation (horizontal spectrum fringes) to restore them and from this compensation the difference of index between solution and glass could be computed. Had there been no difference, the horizontal fringes would have been retained in that part of the spectrum. Such experiments may be made with astonishing ease and accuracy, and I hope soon to communicate quantitative results bearing on this and the preceding paragraph.

[1] *Amer. J. Sci., New Haven*, 31, 1886, (377).

[2] *Proc. Amsterdam Acad.*, 17, 1914, (445); and 18, 1915, (1).

[3] This modification is essential, as the symmetrical apparatus of Michelson aud Morley does not admit of ray separation.

A COMPARISON OF WHITE AND COLORED TROOPS IN RESPECT TO INCIDENCE OF DISEASE

By Lieutenant-Colonel A. G. Love, M.C. U. S. A. and Major C. B. Davenport, Sanitary Corps, U. S. A.

Read before the Academy, November 18, 1918

This paper is an analysis of over half a million admissions to sick report of troops in camps in the United States. Of these 531,445 were for whites and 15,186 for colored troops. The relative frequency of disease in the two 'races' was as 974 to 1155; that is, the colored troops were about 19% more liable to go on sick report than the white troops.

The grounds of sick report are very numerous; so we shall consider only the commoner ones. All comparisons will be of the so-called mean annual rate per 1000 men. The data are those collected for the Report of the Surgeon General of the Army, 1918. Some comparative data have occasionally been included derived from the nine preceding annual reports.

In many cases the morbidity rate is almost the same for the white and colored troops. In this paper particular attention is paid to those diseases that have a strikingly different rate in the two races. It is to be kept in mind that all troops had been subjected to the same examination at induction, in order to exclude chronic diseases; and that they lived under equally good sanitary conditions.

First, may be considered the diseases that are commoner in colored than in white troops. These fall into four groups (a), those in which the excess is due to the fact that fewer of the colored men had become artificially immunized by inoculation before, or at, mobilization. They probably brought their disease to camp; or failed to get a reaction there; (b) those in which there is a lower natural immunity in the negro; (c) venereal disease and its complications, (d) other diseases.

(a) Of the diseases that are due to lack of acquired immunity, smallpox is the most striking. The morbidity rate for this disease is for colored troops 9 times that for white troops (there being 146 admissions altogether). Chickenpox was relatively 8 times as common in colored as in white troops.

(b) Of the diseases for which negroes lack relative natural resistance tuberculosis of the lungs and pneumonia take first places. There were over 2½ times the admission rate for tuberculosis of the lungs in colored as in white troops in 1917. This is a little greater difference than the average of the past

ten years,—where the colored have about 2 times the admission rate of white troops. In tuberculosis of other organs the rate for colored troops is twice that for white.

In both lobar and broncho-pneumonia the morbidity rate of colored men exceeds that of white. For 1917 the rate for lobar pneumonia was 4½ times, for broncho-pneumonia 3 times, as great for colored as for white men. In the ten year record the colored rate is 2.2 times the white rate for broncho-pneumonia. Given, then, troops under similar environment in the presence of conditions favoring pneumonia, there are relatively two to three times as many colored troops as white who show symptoms of the disease.

(c) *Venereal disease.*—In the Army that was mobilized in 1917, the rate for syphilis was about 4 times as great in colored as white troops, for chancroid 4½ times, for gonococcus infection 2½ times. Combining the data of the last ten years the rate for all venereal diseases for colored troops is a little less than double that for whites. The difference between the races in incidence of venereal diseases is probably due partly to a difference in social pressure, partly to a difference in ability to control the sex instinct.

This greater infection of colored troops with venereal disease leads to the greater incidence in that race of various complications of those diseases. Thus, retinitis, iritis, cerebro-spinal meningitis, various diseases of the spinal cord—largely, probably, complication of syphilis—are commoner among the colored soldiers. Similarly, arthritis, osteomyelitis, endocarditis, nephritis and urethritis—complications of gonorrhea—are 2 to 3 times commoner in colored troops than in white.

Of other diseases there are few in which the colored rate is markedly greater than the white. The colored indeed show about 100% more epilepsy and hysteria than the whites, more drug addiction and double the amount of neuralgia and hemorrhoids; and that is about all.

Of diseases that are less common in colored than in white troops there are, first of all, skin diseases. Thus acute abscesses and inflammations of the connective tissue of the skin are about one-third as common in colored as white soldiers; boils are one-fourth as common; dermatitis from traumatisms one-third as common and venomous bites and stings have much less effect. As is well known, the reaction of the skin to cuts differs in the two races. Colored persons tend to form keloid tumors, or skin ridges, along the scar. In civil life deaths from skin cancer are one-fourth as common in colored as white citizens. This by no means exhausts the catalogue of skin differences in the two races.

Also, in the quality of the teeth there is a difference. It is not rare to find negroes who at twenty to thirty years have lost nearly all their teeth through the use of mercury. But, on the other hand, it is much commoner to find negroes than Anglo-Saxons who, at twenty to thirty years, have teeth that show no sign of decay, even where no especial care has been taken of them. The negro teeth are naturally resistant to the organism of tooth caries.

Diphtheria, scarlet fever and German measles are from one-half to one-fourth as common in colored as white men. This experience with morbidity in camps agrees with the census mortality rates of civil life. Thus, for the registration area of the United States (1915) the rates for white and colored respectively are: for scarlet fever, 3.7 and 1.3; for diseases of the larynx, 8 and 4. Also while admissions in camp for measles-complicated-by-lobar-pneumonia are slightly greater in colored than in white troops, cases of measles-complicated-by-broncho-pneumonia are only about half as common in colored troops.

Influenza in the troops during 1917 had a rate of 28 for colored men—about half that for white men (58); and during the past ten years only about one-fourth. It is commonly held that colored children are much less apt to be attacked by the organism of infantile paralysis, which, like influenza, enters the body through the naso-pharynx. Thus, in general, the diseases of the upper respiratory tract are much less common in colored than white people.

In general the skin, not only on the surface of the body but also that which infolded to form the lining of the mouth and naso-pharynx, is much more resistant to micro-organism in negroes than in whites. The white skin seems to be relatively a degenerate skin in this respect.

Moreover, the nervous system of the uninfected negroes shows fewer cases of instability than that of whites. Thus, there is only about one-third as much neurasthenia recorded in camps in 1917; about one-third as many cases of constitutional psychopathic state and one-half as much alcoholism in colored as in white troops. Functional cardiac disturbances of nervous origin are half as common. Eye defects are relatively few, and there is about half as much liability to refractive errors as in whites. Ear diseases are about half as common.

Metabolically, the uninfected colored troops seem less liable to disturbances than white troops. Diabetes is only half as common in the negroes as the whites and about the same is true of urinary calculi. Inflammations of the gall bladder are also much less frequent.

To sum up: The colored troops are relatively less resistant to diseases of the lungs and pleura as well as to certain general diseases, like tuberculosis and smallpox; they are also much more frequently infected with venereal diseases and suffer wide spread complications of these diseases.

But the uninfected negro is highly resistant to diseases of the skin, mouth and throat. He seems to have more stable nerves, has better eyes and metabolizes better. Thus, in many respects the uninfected colored troops show themselves to be constitutionally better physiological machines than the white men.

(Publication approved by the Office of the Surgeon General.)

ADMISSIONS TO SICK REPORT, UNITED STATES TROOPS, FOR YEAR 1917
(From Table No. XV, Report Surgeon General, U. S. Army, 1918)

NUMBERS OF INTERNATIONAL CLASSIFICATIONS	UNITED STATES		
		White	Colored
	Mean strength	545,518	13,150
	Cause of admission to sick report	Ratio per 1000 of mean strength	
	I. Infectious Diseases (Excluding Tuberculosis and Venereal)		
1	Typhoid fever	0.41	0.23
2	Typhus fever	0.01	
3	Relapsing fever (all)	0.05	
4	Malarial fevers (all)	7.50	5.85
4a	Malarial fever, estivo-autumnal	0.20	0.38
4b	Malarial fever, mixed	0.01	0.08
4c	Malarial fever, quartan	0.10	0.08
4d	Malarial fever, tertian	4.10	3.19
4e	Malarial fever, undetermined	3.09	2.13
5	Smallpox	0.22	1.98
6	Measles (all)	83.11	87.07
6a	Measles, uncomplicated	78.96	82.28
6c	Measles with broncho pneumonia	2.48	1.37
6d	Measles with lobar pneumonia	1.25	1.75
6f	Measles with other complications	2.42	1.67
7	Scarlet fever	3.58	0.84
8	Whooping cough	0.04	0.08
9	Diphtheria and results	2.12	0.46
10	Influenza (all)	58.44	27.83
14	Dysentery (all)	0.87	0.84
14a	Dysentery, bacillary	0.09	0.08
14b	Dysentery, balantidic		
14c	Dysentery, entamæbic	0.35	0.30
14d	Dysentery (other protozoal)		
14e	Dysentery, unclassified	0.42	0.46
15	Plague	0.01	
17	Leprosy	0.01	
18	Erysipelas	1.17	1.44
18b	Meningitis, cerebrospinal (epidemic)	1.68	4.11
19a	Chicken pox	0.44	3.42
19b	Diphtheria bacillus carrier	0.45	1.14
19c	German measles	16.36	4.41
19d	Hemoglobinuric fever (nonmalarial)	0.01	0.08
19e	Paratyphoid fever A	0.02	
19f	Paratyphoid fever B		
19g	Trench fever		
19i	Typhoid bacillus carrier	0.02	
19j	Typhoid vaccination	17.65	11.25
19k	Vaccinia	15.69	6.16
19l	Mumps	38.93	37.26

ADMISSIONS TO SICK REPORT—*Continued*

NUMBERS OF INTERNATIONAL CLASSIFICATIONS	UNITED STATES		
		White	Colored
	Mean strength	545,518	13,150
	Causes of admission in sick report	Ratio per 1000 of mean strength	
20	Septicæmia, general	0.09	
20b	Pyæmia, surgical	0.01	
21	Glanders	0.01	
22	Anthrax	0.01	
24	Tetanus	0.01	
25	Trench foot		
25a	Mycoses (others)	0.04	
26	Pellagra	0.04	0.23
27	Beriberi		
47	Rheumatic fever	8.68	6.39
11, 19, 18a, 18c 20a	Infectious disease (others)	2.48	3.19
	II. Tuberculosis (All)		
28	Tuberculosis of lungs	11.69	32.02
29	Miliary tuberculosis, acute	0.07	
30–35	Tuberculosis (others)	0.76	1.90
	III. Venereal Diseases (All)		
37	Syphilis (all)	14.22	54.68
37a	Syphilis, hereditary	0.03	
37b	Syphilis, primary	3.85	20.08
37c	Syphilis, secondary	8.34	22.13
37d	Syphilis, tertiary	2.00	12.47
38a	Chancroid	12.39	52.78
38b	Gonococcus infection	82.47	203.04
	IV. General Diseases (Others)		
36	Rickets	0.12	
39–45	Cancer and other malignant tumors	0.15	
46	Benign tumors	1.17	1.75
48	Arthritis	5.74	10.58
48a	Gout	0.03	
49	Scurvy	0.01	
50	Diabetes mellitus	0.18	0.08
51a	Diseases of the thyroid gland (goiter simple excepted)	1.09	0.38
51b, 52 53b, 116	Diseases of the ductless glands (thyroid excepted)	0.07	0.08
53a	Leukemia	0.01	
54	Anemia	0.20	0.23
55a	Purpura rheumatica	0.01	
56	Inebriety, drug addiction	0.75	1.30
56a	Alcoholism	2.45	1.06
57–59	Poisoning, chronic	0.04	
55, 88	General diseases (others)	5.12	8.14

ADMISSIONS TO SICK REPORT—*Continued*

NUMBERS OF INTERNATIONAL CLASSIFICATIONS	Causes of admission to sick report	UNITED STATES: White	UNITED STATES: Colored
	Mean strength	445,528	13,150
		Ratio per 1000 of mean strength	
	V. Nervous System, Diseases of (All)		
61	Meningitis, simple	0.10	0.23
62	Locomotor ataxia	0.07	
63	Diseases of the spinal cord (others)	0.21	0.53
64	Apoplexy	0.04	
66	Paralysis, cause not specified	0.38	0.46
69	Epilepsy	2.88	6.54
72	Chorea	0.14	
73	Hysteria and results (psychoses excepted)	1.17	2.05
73a	Neuralgia	1.83	3.57
73b	Neuritis	1.13	1.06
74	Neurasthenia	1.50	0.46
60, 65, 70 73, 74a	Diseases of the nervous system (others)	2.21	2.59
	VI. Mental Alienation		
67	General paralysis of the insane	0.05	0.08
68a	Constitutional psychopathic states	0.86	0.30
68b	Dementia precox	1.46	1.75
68c	Mental deficiency	2.80	2.89
68d	Psychasthenia	0.17	
68e	Psychoneurosis	0.42	0.30
56b	Psychoses (alcoholic)	0.25	0.23
68f	Psychoses (nonalcoholic)	0.82	1.37
68	Mental alienation (others)	0.21	0.30
75	*VII. Eyes and Their Annexa, Diseases of*		
75a	Conjunctivitis	7.50	8.44
75b	Trachoma	0.84	0.38
75c	Refractive errors	1.01	0.68
75e	Iritis	0.44	1.06
75f	Keratitis	0.52	0.91
75g	Choroiditis	0.08	
75h	Retinitis	0.10	0.76
75i	The eyes and annexa (other diseases of)	2.71	5.40
76	*VIII. Ear, Diseases of*		
76a	Mastoiditis	0.45	0.15
76b	Otitis media	10.64	5.48
76c	Ears, other diseases of	0.83	0.91
86	*IX. Nasal Fossæ, Diseases of*		
86a	Adenoids	0.38	0.30
86b	Deviation of nasal septum or spurs	2.04	0.30
86c	Sinusitis	1.49	1.60
86d	Nasal fossæ, other diseases of	13.16	10.04

ADMISSIONS TO SICK REPORT—*Continued*

NUMBERS OF INTERNATIONAL CLASSIFICATIONS	UNITED STATES		
		White	Colored
	Mean strength	545,518	13,150
	Causes of admission to sick report	Ratio per 1000 of mean strength	
	X. Throat, Diseases of		
87	Larynx, diseases of	8.38	4.56
100	Pharynx, diseases of	14.54	12.47
100a	Tonsils, diseases of	77.64	50.95
	XI. Circulatory System, Diseases of		
77	Pericarditis	0.08	0.08
78	Endocarditis, acute	0.15	0.53
79	Valvular heart diseases	4.21	6.08
79a	Heart, other organic diseases of	1.94	4.56
80	Angina pectoris	0.13	0.35
81a	Aneurysm	0.02	
81	Arteries, other diseases of	0.13	0.15
82	Embolism and thrombosis	0.05	
83a	Hemorrhoids	6.55	14.60
83b	Varicocele	3.72	2.51
83	Veins, other diseases of	1.89	1.67
84a	Status lymphaticus		0.15
84	Lymphatic system, other diseases of	4.07	5.86
85a	Cardiac arrhythmias	0.06	
85b	Cardiac murmur, not organic	0.03	0.08
85c	Cardiac disorders, functional	0.19	0.08
85d	Hemorrhage	0.48	0.68
85	Circulatory system, other diseases of	1.08	2.36
	XII. Respiratory System, Diseases of		
89, 90	Bronchitis	74.10	61.75
91	Broncho-pneumonia	2.51	7.30
92	Pneumonia, lobar	11.47	53.69
92b	Pneumonia, unclassified	0.08	0.23
93	Pleurisy	5.17	9.43
95	Gangrene of lung	0.01	
98	Abscess of lung	0.01	
96, 97	Respiratory system, other diseases of	2.17	4.03
	XIII. Digestive System, Diseases of		
99	Mouth and annexa, diseases of	3.91	4.18
101	Esophagus, diseases of	0.01	
102	Ulcer of the stomach	0.49	0.30
103	Stomach, diseases of (ulcer and cancer excepted)	10.68	9.73
106	Ankylostomiasis	0.58	
107a	Intestinal parasites, others	0.41	0.38
108	Appendicitis	9.83	6.31
109	Hernia	8.98	10.57

ADMISSIONS TO SICK REPORT—*Continued*

NUMBERS OF INTERNATIONAL CLASSIFICATIONS	UNITED STATES		
		White	Colored
	Mean strength	545,518	13,150
	Causes of admissions to sick report	Ratio per 1000 of mean strength	
109a	Intestinal obstruction	0.15	0.23
105, 110	Intestines, other diseases of	47.35	33.76
112	Hydatid tumor of liver		
113	Cirrhosis of the liver	0.05	
114	Cholelithiasis	0.08	
115	Cholecystitis	0.87	0.38
115a	Liver, other diseases of	1.97	0.61
117	Peritonitis, acute diffuse	0.02	
117a	Peritonitis, acute local	0.02	
117b	Peritonitis, chronic	0.01	0.08
117c	Peritoneal adhesions	0.37	
118	Other diseases of the digestive system	0.23	0.30
118a	Pancreatitis		
	XIV. Genito-Urinary System and Annexa, Nonvenereal Diseases of		
119	Nephritis, acute	0.47	0.76
120	Nephritis, chronic (Bright's disease)	0.71	0.91
122	Kidney and annexa, other diseases of	0.72	0.91
123	Calculi of urinary passages	0.43	0.23
124	Bladder, diseases of	3.24	4.41
125	Urethra, diseases of	0.83	1.37
126	Prostate, diseases of	0.19	0.15
127	Male genital organs, diseases of (nonvenereal)	4.47	6.54
	XV. Skin and Cellular Tissue, Diseases of		
142	Gangrene	0.03	
143	Furuncle	8.78	2.13
144	Acute abscess	5.83	2.89
144a	Cellulitis	4.52	1.14
145	Skin and annexa, other diseases of	17.29	11.86
	XVI. Bones and the Organs of Locomotion, Diseases of		
146	Osteomyelitis	0.36	0.61
146a	Bones, other diseases of (tuberculosis excepted)	0.93	1.44
147	Joint, diseases of (rheumatism and tuberculosis excepted)	1.59	1.98
149	Pes planus	8.40	9.28
149a	Organs of locomotion, other diseases of	10.71	9.89
	XVII. Congenital Malformations and Ill-Defined Diseases		
150	Congenital malformations	8.23	12.09
189	Not specified or ill-defined diseases	36.17	51.41
	Total for diseases	882.54	1064.41

ADMISSIONS TO SICK REPORT—*Continued*

NUMBERS OF INTERNATIONAL CLASSIFICATIONS	UNITED STATES		
		White	Colored
	Mean strength	545,518	13,150
	Causes of admission to sick report	Ratio per 1000 of mean strength	
Office classification	*Traumatism Produced by External Causes* Total	91.68	90.42
03	Bites	0.13	
04	Burns	1.54	1.06
05, 06	Burns, chemical and X-ray	0.34	0.08
09	Compression (brain)	0.01	
10	Concussion (brain)	0.63	0.23
12	Contusion	3.41	0.68
14	Crushing	0.43	0.23
17–20	Dermatitis from traumatism	2.23	0.68
21	Dislocation of joint	1.84	1.52
62	Dislocation, articular, cartilage of knee	0.16	
22, 23	Drowning (except suicides)	0.04	
28	Epiphyseal separation	0.01	
08, 11, 13, 32, 33, 68, 69	Eye and annexa, traumatisms	0.34	0.46
34	Foreign bodies	0.25	0.30
36	Fracture, simple	7.63	4.41
35	Fracture, compound	0.52	0.46
38–48	Fracture, faulty union	0.13	0.23
0x	Fracture near joint, with dislocation	0.02	
37	Fracture, comminuted	0.22	0.30
50	Gases deleterious, absorption of	0.03	
51	Gunshot wound	1.02	1.44
552–56	Hemorrhage and results	0.09	
59	Homicides	0.01	0.23
66	Poisoning by food	1.04	0.61
67	Poisoning, other acute	0.22	0.08
96	Secondary result of injury	5.55	11.79
73	Sprain of joint	18.69	16.73
75	Strain	2.88	3.19
79	Suicide	0.05	0.15
81	Synovitis, traumatic, cause undetermined	0.97	0.91
85	Venomous bites and stings	0.31	0.08
93	Wounds, multiple	0.07	0.08
94	Wounds, penetrating	0.16	0.53
92	Wounds, extensive	0.01	
95	Wounds, perforating	0.90	1.82
86–91	Wounds, others	30.62	32.02

ADMISSIONS TO SICK REPORT—*Concluded*

NUMBERS OF INTERNATIONAL CLASSIFICATIONT	UNITED STATES		
		White	Colored
	Mean strength	545,518	13,150
	Causes of admission to sick report	Ratio per 1000 of mean strength	
00–02, 07, 15, 16,24–27,29–31,49, 57,58, 60,61, 63–65, 70–72, 74,76–78,82–84,97–99	Traumatisms, others	9.14	10.04
80	Sunstroke	0.04	0.08
	Total for diseases	882.51	1064.41
	Total for external causes	91.69	90.42
	Grand total	974.19	1154.83

PERTURBATIONS AND TABLES OF THE MINOR PLANETS DISCOVERED BY JAMES C. WATSON

BY ARMIN O. LEUSCHNER

BERKELEY ASTRONOMICAL DEPARTMENT, UNIVERSITY OF CALIFORNIA

Read before the Academy, April 16, 1916

Among the many important contributions to Astronomical Science by James C. Watson, one of the original members of the National Academy of Sciences, is the discovery of twenty-two minor planets commonly known as the Watson asteriods. The first of these, (79) Eurynome, was discovered at Ann Arbor on September 14, 1863, and the last, (179) Klytaemnestra, on November 11, 1877, three years before his death. By his will a fund was bequeathed in trust to the National Academy of Sciences for the purpose of promoting astronomical research. One of the objects specifically designated was the construction of tables of the perturbations of the minor planets discovered by the testator. From the beginning Prof. Simon Newcomb was a

member of the board of Watson Trustees; later he became chairman of the board, and remained in that capacity on the board until his death in 1908, being succeeded by Prof. E. C. Pickering, whose associate trustees now are Professors E. B. Frost and W. L. Elkin. The National Academy of Sciences through its various boards of trustees has at all times made the most persistent efforts to discharge the chief obligation imposed by the trust, by providing tables of the perturbations of the Watson asteroids.

My own connection with this great undertaking is best explained by the following substance from a letter addressed to me by Professor Newcomb under date of June 4, 1901.

> I have consulted with the Watson Trustees on the subject of having the perturbations of certain Watson asteroids computed under your direction. We are all favorable to the project provided we can have some assurance of success. We have had the coöperation of many capable men but so far the result has not been satisfactory. The general outcome of the matter is that we are supplied with a mass of papers, computations, perturbations, and perhaps tables, but nothing has yet reached us in a form complete and perfect for publication as tables. We had just about reached the conclusion that the work must be done here at Washington under my personal direction and according to formulae which I should supply.

For further details in regard to the earlier history of the work I must refer to the introduction written by Professor Newcomb in March, 1908, to the tables of twelve Watson planets completed under my direction in 1907 and published in 1910 as volume X, Seventh Memoir of the National Academy of Sciences.

Actual work was commenced by myself with the assistance of Drs. Russell Tracy Crawford and Frank Elmore Ross according to a somewhat definite program carefully arranged by Professor Newcomb. Our investigations were to be confined to those thirteen planets, for which so far no attempt for derivation of the perturbations had been made, Professor Newcomb's purpose being to have the difficulties encountered with the remaining planets investigated under his personal direction at Washington. One planet, (132) Aethra, was to be entirely excluded from the program, having remained lost since the year of its discovery in 1873. But in April, 1903, it was agreed that all the previous investigations should be turned over to me. At the same time it had become apparent that the success of the undertaking demanded that I should be given the utmost freedom in planning and conducting the investigations, to which the trustees readily consented.

Today it is my great privilege to be able to report to you, that except for the completion of the manuscript for publication, and some minor numerical work, the task is accomplished and that the obligations of the National Academy under the Watson Trust pertaining to the perturbations and tables of the minor planets discovered by Watson, will soon be fully discharged.

It is impossible in this brief space of time to give a complete history of the work, including the part taken by my numerous assistants. I must also refrain from reporting on the many scientific results attained as to methods of attack

and solution, as these involve mathematical detail. I can touch here only on a few points which must serve to illustrate the general character of the work. But first of all it would seem necessary briefly to review what objects are to be served in determining perturbations.

When a planet moves solely under the central attraction of the Sun it describes an ellipse defined in space by six elements or constants, of which for our present purposes three are of prime importance, namely the mean daily motion, or briefly, mean motion, which is the average angular velocity about the Sun, or 360° divided by the period of revolution, in days; the eccentricity of the ellipse; and the inclination of the plane of its motion to the ecliptic. The six elements at any instant depend on the three coördinates defining the position of the planet at the instant and on the three components of velocity. When the constant elements are ascertained and the body continues to move solely under the Sun's attraction, the position and velocity of the planet may be obtained from them at any other time and vice versa. Such positions and velocities are called undisturbed positions and velocities. But if to the Sun's attraction is added that of one or more major planets, like Jupiter, then the velocity is changed and the planet will depart from its undisturbed ellipse. These departures are called perturbations. From a disturbed position and velocity at any instant elements may again be determined for that instant. They represent disturbed elements, as compared with the former. Thus different elliptic elements may be computed for different instants from the corresponding positions and velocities attained under the attraction of the Sun and major planets. These are called osculating elements. Any one set may be adopted as undisturbed and the differences of the others from that set represent the perturbations in the elements.

Whether the perturbations are expressed in the elements or in the coördinates, they are determined by the integration of differential equations, the solution of which in the present state of astronomical science involves expansion in trigonometrical series. The integration introduces divisors which become very small, when the mean motion of the minor planet is in a commensurable ratio to that of a disturbing planet. Therefore since Jupiter's mean motion is 299″ or nearly 300,″ such series fail for instance for a planet with mean motion nearly 300″ or 600″ or 900″. Since Hansen's method, to the application of which our work was originally to be restricted, is based on series of the character referred to, it was bound to fail for planets with a commensurable ratio. This was our actual experience with the first planet undertaken.

This difficulty, however, is purely a mathematical one and has been overcome by Bohlin in his "Gruppenweise Berechnung der Stoerungen" for the group 1/3 by the introduction of the simple expedient of using the exponential for the trigonometrical form in the series. In fact he has published tables for the exact commensurability 1/3 in which all of the elements appear explicitly in the coefficients. The series referred to progress according to

powers of the eccentricities of the disturbed and disturbing planets and of the mutual inclination of their orbit planes. In order to avoid unmanageable expressions Bohlin had to confine himself to terms of the second and sometimes third degree in these quantities, while Hansen's method with the aid of Bessel's functions for the eccentricities imposes no restrictions in this respect.

Bohlin's method otherwise closely resembles Hansen's in the treatment of the perturbations, the only other distinctive feature being that for planets with mean motion nearly commensurable with Jupiter's, the perturbations are developed by Taylor's theorem in ascending powers of a quantity w which depends on the difference between a multiple of a planet's actual mean motion and a multiple of Jupiter's so that ultimately series within series progress according to powers of the mass of the disturbing body, of the eccentricities of the disturbed and disturbing bodies, of the mutual inclination of their orbit planes, and of the quantity w.

Now several of the Watson planets belong to the Hecuba Group or Group 1/2, having a mean motion of nearly 600″ or about twice that of Jupiter, and as no theory then existed for this group it was decided to develop the theory and tables for this group 1/2 on Bohlin's plan for the group 1/3. But owing to the great complexity of the problem and the intricate transformations involved it was thought wise first of all to assure ourselves of an exact understanding of Bohlin's method by reproducing selected values of his tables for the group 1/3. In this we failed in many instances. After much fruitless search for the cause of the discrepancies these were called to the attention of Bohlin, who promptly replied that he had become aware of inaccuracies in his theory and tables and that he had already completed a revision of his work, sending at the same time advance proof sheets, verifying our conclusions. We now felt safe in attacking the mathematical theory of group 1/2 and after another year's work on the theory and tables of group 1/2 preparatory to the application of Bohlin's method to the Watson planets of that group, we learned from Bohlin that von Zeipel was engaged in the same task. A little later we received von Zeipel's printed tables. These we at once compared with our own, many transformations from one to the other being necessary on account of the difference of developments used, but to our dismay we discovered many disagreements. By correspondence these have practically all been cleared up and thanks to the careful system of checks adopted in our work we found it unnecessary to change any of our results. In some minor respects we still differ, but the expressions on which the numbers in question are based are so complicated that von Zeipel doubts whether he can remember how he has obtained his values. We are thus abiding by our own results, which have been fully verified as I shall show a little later.

The mathematical and numerical work involved in the revision of von Zeipel's theory has been performed under my direction by Miss Anna Estelle Glancy and Miss Sophia H. Levy. The former has also prepared a complete

set of directions for the application of von Zeipel's theory for the Hecuba Group and has illustrated the same with full numerical details for his example, 10 Hygiea, for the guidance of computers.

The work to which I have just referred serves to indicate how the original program of routine computation to the first order of mass with respect to Jupiter by Hansen's established formulae was completely changed to include extensive theoretical investigations.

Although Hansen's and Bohlin's methods—the latter extended by von Zeipel and ourselves, and also by D. F. Wilson, to provide tables for groups 1/2, and 5/2 in addition to group 1/3—were entirely sufficient to master the Watson planets, for which, however, the group 5/2 was not applied, an extensive theoretical analysis has also been made of Brendel's recent methods based on the researches of Gyldèn. These methods appeared while our work was well under way. In passing I might state that all this theoretical work was done independently of the Watson funds.

Aside from these purely theoretical problems the Watson work naturally falls under two heads:

(1) The investigation of the causes of the failure of the earlier work as referred to by Newcomb.

(2) The application of the most suitable methods to the planets on which work had not been undertaken.

In addition the ultimate investigation of the cause of the loss of (132) Aethra was constantly kept in mind.

Under the first head I wish to make especial reference to the work done by Eichelberger on (93) Minerva. Professor Newcomb naturally suspected an error of computation on account of an apparent motion of the node indicated by the outstanding discrepancies in the representation of the observations. Newcomb himself attempted to correct Eichelberger's investigations by further computations at Washington. But later I found the whole trouble to be due to the fact that too much had been expected from first order perturbations by Jupiter, that is, perturbations involving only the first power of the mass. I suggested that a repetition of the work on Minervà to only four or five places would accomplish as much as the previous investigation to seven figures and that the residuals of the observations would reveal the same implied motion of the node; that the latter was of a higher order; and that it could be determined empirically either from our own or Eichelberger's outstanding residuals. Professor Newcomb welcomed this test and all of Eichelberger's work was repeated under the plan proposed, but on the basis of mean instead of Eichelberger's osculating elements and with the mean instead of the eccentric anomaly as independent variable, with the result that the suspected cause of the trouble was fully substantiated. The outstanding discrepancies which were really negligible under the original program but appeared large because of the great accuracy with which Eichelberger had computed, were then removed by an empirical determination of the still remaining motion of the node.

At first Professor Newcomb was greatly surprised by the considerable difference between our coefficients of the perturbations and those of Eichelberger, but formulae were soon derived which enabled me to deduce one from the other on the basis of the difference in elements used and thus to verify both sets of developments. After this investigation Eichelberger's original tables were published in an abridged form.

More serious complications were encountered in the revision of other computations, which were due to misconceptions of theory and numerical errors.

The integration of the differential equations for the perturbations gives rise to certain constants in the developments, to terms multiplied by the time (secular terms), and to purely trigonometrical or periodic terms. All of these added to the undisturbed coördinates give the disturbed place. Some of these terms are of the same form as terms employed in computing the undisturbed positions and may be combined with them or introduced in the elements. It then becomes merely a matter of arbitrary classification or combination, as to what part of the disturbed coördinate shall be directly computed from properly changed elements and what part shall be left over as perturbations. When all but the purely periodic terms are put into the elements, we may deduce a sort of mean elements, and we can thus speak also of a mean mean motion. These elements involve properly chosen constants and are not of the sort that they represent the actual position and velocity at any particular time. It was found that some of my predecessors in the work had misinterpreted the terms they were dealing with although their developments were numerically correct.

The tracing of these and other inaccuracies for the planets to be revised have absorbed more time and effort than all the work on the planets that had not been previously undertaken. To simplify the determination of the constants and to secure greater accuracy Professor Newcomb's program originally called for adopting the mean of the several sets of elements printed for each planet in the B. J. as approximate mean elements, but later we shall see that in the case of Andromache the actual mean elements lie outside of the osculating elements obtained from observations and special perturbations over a period of thirty-three years. Thus we actually at first started with elements which were neither osculating nor mean elements, which involved corresponding difficulties later. But it should not be forgotten that at that time the whole problem of proper attack was still in an experimental stage.

With reference to Brendel's very exhaustive and brilliant methods I may remark that I have come to the conclusion that even if they stand the test of the very critical cases of near commensurability, for practical purposes they do not have the advantage of the same simplicity as a proper choice between Hansen's and Bohlin's methods.

For six of the twelve sets of tables published in 1910 comparison of observations taken in 1912–13 have been made with theory by Miss A. R. Kidder, with an agreement of less than the proposed 1′ of arc, excepting in one

case, (133) Cyrene, which although approaching the Hecuba type had been developed by Hansen's method in the early stages of our investigations. As the observation used in this case, however, is only approximate, a further test may show the slight discrepancy to be erroneous. These comparisons represent a remarkable result for the published tables, as in these cases some twelve years had elapsed since the last opposition used for the correction of the elements and for the tables.

In conclusion I may refer briefly to two of the most interesting Watson planets.

The Watson planet which promises to become one of the most interesting of the twenty-two under consideration, if not of all the 1400 or more so far discovered, is (175) Andromache, which belongs to the Hecuba group, or group 1/2, the mean daily motion being approximately twice that of Jupiter. Andromache was found by Watson on October 1, 1877 and was the last but one discovered by him, his latest discovery, as stated before, being (179) Klytaemnestra on November 11, 1877. Andromache was assiduously observed for one month, 43 observations at widely scattered observatories being secured by the most competent observers. The calculation of the orbit yielded a mean motion of approximately 550″. In spite of diligent search the planet was not found again from any predictions based on the original elements. This failure to find Andromache is a striking illustration of the fallacy of some of the orbits included as thoroughly reliable in our published lists. On May 19, 1893 a presumably new planet was discovered at Nice and given the provisional designation 1893 Z. From observations extending over more than two months an orbit resulted resembling the orbit of Andromache, except for the mean motion which was found to be at first 617″ and later 610.″

The following possibilities existed then with reference to Andromache and 1893 Z. (1) the planets might be identical, the adopted mean motion of 550″ of Andromache being erroneous. (2) The planets might be identical and the mean motion might have changed from the smaller value of 550″ to the larger value of 610,″ passing through 598″ which is twice the mean motion of Jupiter. If this were true a phenomenon of the utmost importance to science, known as libration, would come under consideration. (3) The planets might not be identical.

A study of the observations of Andromache revealed an uncertainty of the comparison star of one of the fundamental positions. From the orbit of 1893 Z positions were then calculated for the dates of observation of Andromache in 1877 and a satisfactory agreement found. The identity of the two planets was thus established in 1893–94. At the same time the hope for the occurrence of the first case of libration for a planet of the group 1/2 was shattered, for the mean motion in 1877 turned out larger by several seconds than the 610″ calculated for 1893. On the basis of these computations Andromache was also found on some Heidelberg plates taken in 1892.

The close commensurability of the mean motion with twice Jupiter's mean motion made it apparent that a satisfactory determination of the perturbations of this planet by Hansen's method was entirely out of the question, in fact impossible. After the revision of von Zeipel's tables for the Hecuba group, to which I have already made reference, this, our most difficult and striking case of the Hecuba group, was selected as a test case for our tables. After applying our perturbations, the least squares correction was based on ten oppositions from 1894 to 1907 with eminently satisfactory results, not only for these thirteen years but also for the 1877 oppositions, seventeen years before, and for a subsequent opposition in 1911. According to Miss Sophia H. Levy's computations, the perturbations of this planet are the largest we have experienced, the coefficient of the largest term in the perturbation of the mean anomaly reaching nearly 30°. For the 1877 opposition the perburbation in the mean anomaly is 24°. These amounts generally produce double the displacement in geocentric position at opposition and yet an approximate right ascension and declination published by Wolf in 1911 is represented by our theory to within 4′ of arc in right ascension and less than 1′ in declination. A further very slight revision of the theory is contemplated before publication to include some terms of higher degree depending on the very large eccentricity of 20°. This revision will make this representation still more satisfactory, although the larger part of all outstanding differences is due to perturbations by Saturn.

Even then our representation is much closer than was to be expected. Another approximate position has recently been published by Wolf for January 16, 1914. This is equally well represented, showing that the theory has definitely been verified by observation. The importance of this result lies in the fact, that with the most difficult case of the Hecuba type conquered, the revised tables of von Zeipel now provide a ready and accurate means of representing the motions of all planets of this type at present known. Confirmation of this statement is afforded by our subsequent work on (104) Clymene, (106) Dione, and (168) Sybilla of the same group with mean motions of 634″, 629″, and 572″.

But this planet has another striking significance. To prevent its loss, pending the computation of its theory under the Watson Fund, Berberich of the Kgl. Rechensinstitut, Berlin, has applied the laborious process of special perturbations by Jupiter and Saturn for the determination of osculating elements. The extent of his unpublished work was not known to me, until some years ago I addressed an inquiry to Berlin in regard to any unpublished data, particularly the Saturn perturbations.

Among the unpublished data are osculating elements for epochs in 1877, 1892 and from then on for practically every year until 1910. According to these elements the mean motion in 1877 was 617″.7. In 1892, fourteen and a half years later, it was 614″. For three and a half years it oscillated about this figure and then in another fourteen and a half years to 1910 gradually

diminished to 608″.2, the last figure available. Thus in 1910 Hecuba had already encroached well within the previously known gap about 600″.

In von Zeipel's theory a transformation of the given osculating elements is made at the outset with the aid of the general tables for the group 1/2. The resultant elements which are retained as constants until the theory is completed, are not exactly mean elements in the sense defined above, but they approximate the same. Since from 1877 to 1910, or in thirty-three years, the osculating mean motion has gradually diminished by approximately 10″, it might be supposed that our own approximate starting mean mean motion would lie somewhere within this range, or at least be less than the original maximum value of 617″.7 in 1877, but as a matter of fact our approximate mean mean motion is larger by 1″.8. The numerical developments indicate that the mean motion may continue to diminish. Whether the mean motion of 598″ will actually be passed, will form a significant problem of research in the theory of motion of minor planets. But the following facts stand out prominently even now: (1) The extent of the hitherto known gap about 600″ has been greatly diminished; (2) stability has not been impaired thereby as our representation of the 1914 observation shows, for which the perturbation in the mean anomaly has now diminished from the original 24° in 1877 to a comparatively small quantity. (3) E. W. Brown's conclusion stated in his vice-presidential address before the Section A of the A. A. A. S. that if instability exists at or near 598″, it must be for mean motion exceedingly close to that figure has been verified, and (4) if Andromache's mean motion should pass through 598″ stability will probably not be impaired, because our developments indicate no discontinuity for that condition.

Another planet to which I desire to refer is the lost planet Aethra. From investigations by Dr. Dinsmore Alter the cause of the loss of Aethra is due to the fact that the elements on which extensive search has been based for several decades are wholly unreliable, although published to seven figures. In fact the observations can be represented by orbits ranging in mean motion from 800″ to 900″ and more. All that can be said about this planet at the present time is that it may be anywhere at any time within a belt of the celestial sphere 2°20′ wide and that its rediscovery and identification will be a matter of chance, unless it is located on the supposition that an unknown planet discovered in 1913 at the Lowell Observatory, but observed only once, was Aethra. An orbit representing all the original observations of Aethra from June 13 to July 5, 1873, as well as the position of the unknown object in 1913 would make the average mean motion in the interval of forty years equal to about 883″. Details of the investigation and a search ephemeris extending to March 1 of this year, were published in a Lick Observatory Bulletin in November, 1915, but war conditions in Europe and unfavorable weather in California seem to have prevented adequate search. (An exhaustive, but unsuccessful, search has since been reported by Dr. Anna Estelle Glancy in *Astronomical Journal*, **31**, No. 723, April, 1918.)

With reference to the other Watson planets it may safely be asserted that excepting the need of further observations of Andromache for reasons stated above, all of these planets may be stricken from any observing program for decades to come, since they may readily be identified if science so demands, on the basis of the orbits and perturbations determined under the auspices of this academy.

CONGENITAL ARTERIO-VENOUS AND LYMPHATICO-VENOUS FISTULAE. UNIQUE CLINICAL AND EXPERIMENTAL OBSERVATIONS.

By William S. Halsted

Medical School, Johns Hopkins University

Read before the Academy, November 18, 1918

A. Advance of proximal arterial dilation conforming to shifted position of fistula.—Thanks to the assistance of highly competent secretaries I have abstracts of about 400 cases of arterio-venous fistula. These have been studied with special reference to occasional observations on the dilation of the arteries. In 52 instances proximal dilation of the arterial trunk has been noted. I am quite sure that in almost every instance in which the fistula had existed for two or more months proximal dilation of the artery would have been demonstrable if looked for.

Congenital arterio-venous fistula is rare, particularly so when unassociated with naevus. We have been able to find reports of only 3 cases without and 6 with naevus. Of the former none has been cured, unless we except a case (von Eiselsberg's) in which an attempt to cure a fistula between the popliteal artery and vein was followed by gangrene, necessitating amputation of the thigh.

The following case, unique in several particulars, is reported to record the arterial changes observed at 2 operations, the second performed six and one-half years after the first. The patient, a girl aged eleven years, was operated upon November 15, 1911, for a congenital arterio-venous fistula below the angle of the jaw on the right side. After the removal of enormously dilated veins (lantern slide) it was found that the fistula was between one of these and the external carotid artery near the bifurcation or ventricle of the common carotid. Fortunately a careful note was made of a very small anomalous, ascending branch given off from the external carotid just proximal to the fistula (lantern slide). There was great dilation of the common carotid artery and of the external carotid proximal to the fistula, whereas the internal carotid was surprisingly small (lantern slide). The vessels concerned in the fistula formation were excised, the aberrant artery happily being spared. The child was relieved by the operation of very distressing symptoms, but a few

weeks later signs of a second, smaller fistula developed at a distal point just below and in front of the ear. A second operation, proposed frequently, was not acceded to until last spring, six and one-half years after the first. At this operation, performed by Dr. Mont Reid in my presence, remarkably interesting observations were made. The tiny aberrant artery had become dilated almost to the size of a goose quill, and the internal carotid which at the first operation was strikingly small, was as large as normal, or larger (lantern slide). The explanation of the sequence of events is, I think, clear. There were originally two fistulae. The chief of these being eliminated at the first operation, the second, distal to the first, functioned more and more freely in the course of the six and one-half years. The internal carotid, small at the first operation because it was central to the main fistula, became dilated after the subordinate or distal fistula became active; and the anomalous artery, also central to the main fistula, became greatly dilated consequent upon the shifting of the fistula to a distal position.

B. Enlargement of the heart in casès of arterio-venous fistula and persistent ductus arteriosus.—A particularly interesting result of our clinical and experimental studies of arterio-venous fistula is the discovery that enlargement of the heart probably occurs, after a time, as a rule in the major cases. For ten years or more we have noted the condition of the heart in our patients with arterio-venous fistula and have, I believe, quite invariably found the heart enlarged—strikingly so in several instances. Dr. Mont Reid, of our Surgical Staff, has in preparation a report upon his experimental and clinical work on arterio-venous fistula in which he will offer convincing proof of our view that the fistula in its consequences may profoundly affect the heart as well as the veins and arteries. The skiagraphs (exhibit) show the effects of a fistula made three and one-half years ago by Dr. Reid between the carotid artery and external jugular vein of a dog. The veins of the neck on both sides are dilated and the carotid artery is dilated central to the fistula. The heart after two years shows slight enlargement, and after three years, as you observe, it has become pronouncedly increased in size. If the assumption is correct that the heart dilates in consequence of arterio-venous fistula it is important that the fact should be brought to the attention not only of surgeons but also of pathologists and internists who evidently have overlooked it.

Our experimental and clinical observations on arterio-venous fistula and partial occlusion of large arteries may ultimately aid in the explanation of the sequelae of certain congenital anomalies of the heart and aorta. May we not regard the persistent ductus arteriosis as an arterio-venous fistula, the pulmonary artery and the right heart representing the venous side of the fistula? The enlargement of the left heart we might assume to be analagous to the dilation of the artery proximal to a fistula; and in the dilation of the right heart and pulmonary artery we recall the dilation of the veins. My studies on the subject of the dilation of an artery which we find occurs distal to a constricting metal band and distal to the compression exercised by a cervical rib have

led me to investigate the results of the congenital coarctations of the aorta at or beyond its isthmus. I have been interested to find that in a large percentage of these cases of coarcted aorta there is dilation beyond the site of the coarctation. The generally accepted view that this dilation is to enable the aorta to better carry on the anastomotic circulation must, it seems to me, be erroneous. When we shall have ascertained the cause of the arterial dilation obtained experimentally below constricting bands and of the dilation of the artery proximal to an arterio-venous fistula we may be able to explain the dilation of the aorta beyond the congenital coarctation.

C. Plausible explanation of the presence of blood in lymph-cysts at the second and subsequent tappings.—A few years ago, assisted by Dr. Heuer, I removed from the abdomen of a woman about forty years of age a huge congenital hygroma or lymph-cyst. The diaphragm was pushed high up into the right thorax and the liver was displaced far to the right and so rotated on its vertical axis that its inferior border instead of being transverse was parallel and almost in line with the linea alba. The enucleation of the greater part of the cyst was easily accomplished, the few adhesions being disposed of by gentle, blunt dissection. Finally, when there remained only a few filamentous fibers binding the sac to the right adrenal gland[1] and the inferior vena cava we proceeded with even more deliberation and caution. The adhesions to the vein were so delicate that the gentlest manipulation with the handle of the scalpel sufficed to break them. We had an unusually free and clear exposure of the vein and were operating without embarrassment. Suddenly blood gushed from a linear defect about 3 mm. long in the vena cava. The hemorrhage was promptly controlled and the slit in the vessel sutured. Proceeding thereafter with perhaps even greater delicacy, we were again confronted with a gush of blood from the vena cava at a higher point. Here we found a slit from 1.5 to 2 cm. long in this vein. The edges of the slit were smooth, the linear defect being clearly not due to a tear or cut. The gap in the vein was closed by suture. Dr. Heuer and I satisfactorily assured ourselves that there was no defect or special thinning of the wall of the cyst at the point contraposed to the larger of the two defects in the wall of the vena cava.[2]

The slits in the wall of the vena cava were surely not artefacts. They represented, I believe, imperfectly closed embryonic communications between the vein and lymph buds or lymphatic vessels. Dr. Florence Sabin to whom we owe so much for our knowledge of the origin and development of the lymphatic system very kindly writes me in regard to this case as follows:

> Recent work on the lymphatic system serves to demonstrate that lymphatic vessels are modified veins. It has been shown that lymphatic vessels occur first in the neck as sacs, lined with endothelium and packed with blood, which lie close to the jugular veins. The abdominal lymphatics begin as a sac which lies close to that part of the inferior cava which connects the two Wolffian bodies. Baetjer showed in 1908 that in the pig this sac which is the forerunner of the retroperitoneal lymphatics communicates for a time with the inferior vena cava. These communications between the lymphatics and the abdominal veins which are transitory in the pig were then shown to be permanent in the South American monkeys

by Silvester in 1912 (lantern slides), while in 1915 Job demonstrated similar permanent connections in rodents. Thus the study of the development of the lymphatic system affords an explanation of anomalies involving connections between the lymphatic vessels and both the renal veins and the inferior vena cava.

The statement has repeatedly been made that hygromata which at the first tapping have yielded a clear fluid may be found at all subsequent tappings to contain more or less blood. Only one explanation has been offered for the presence of the blood, viz., trauma of the wall of the cyst. This explanation has always seemed to me an unsatisfactory one because the walls of these cysts are as a rule very thin and non-vascular. May it not, in view of the findings in our case, be possible that vestigia of lymphatico-venous communications are responsible for the admixture of blood which has occasionally been noted at only the second and subsequent tappings of lymph-cysts and is more frequently found at the first tapping? The negative pressure consequent upon the aspiration of fluid from the cyst might divert for the moment a little blood from the vein which had given origin to the hygroma's lymphatic bud or vessel. Thus the contents of the sac, clear at the first withdrawal, would be blood stained at the second. Thereafter, with each tapping blood would be aspirated into the sac and hence clear fluid might never again be obtained.

[1] The relation of the cyst to the right adrenal gland was remarkable. In the course of stripping the sac's final delicate attachments we exposed a flat, black surface, evidently the spread-out medulla of the adrenal, about the size of a half dollar. Parenchymatous oozing from this surface required for its arrest a few mattress sutures of fine silk.

[2] The patient recovered promptly and has enjoyed excellent health since the operation.

TABLES OF THE ZONAL SPHERICAL HARMONIC OF THE SECOND KIND $Q_1(z)$ AND $Q_1'(z)$

By A. G. Webster and Willard Fisher

Ballistic Institute, Clark University*

Communicated, January 9, 1919

In connection with certain investigations that I was asked to make with regard to submarines it seemed convenient to have tables of the zonal harmonic of the second kind. These are desirable in various problems in mathematical physics connected with ellipsoids of revolution, such as problems in magnetism. Failing to find such tables in the literature of the subject, I requested Dr. Fisher to calculate tables of the function of the first order and its derivative for an interval that will be sufficient for most practical problems. Dr. Fisher has now left the country, and the stress of other work that prevented their publication being over, I trust that they may be available for peaceful purposes.—A. G. W.

* Contribution from the Ballistic Institute, Clark University, No. 1.

$$Q_1(z) = \frac{z}{2}\log_e\frac{z+1}{z-1} - 1.$$

z	0	1	2	3	4	5	6	7	8	9
1.10	0.6745	0.6708	0.6671	0.6635	0.6600	0.6565	0.6530	0.6495	0.6461	0.6428
1.11	.6394	.6362	.6329	.6297	.6265	.6234	.6203	.6172	.6141	.6111
1.12	.6081	.6052	.6023	.5994	.5965	.5937	.5909	.5881	.5854	.5826
1.13	.5799	.5773	.5746	.5720	.5694	.5668	.5643	.5618	.5593	.5568
1.14	.5543	.5519	.5495	.5471	.5448	.5424	.5401	.5378	.5355	.5332
1.15	.5310	.5288	.5266	.5244	.5222	.5200	.5179	.5158	.5137	.5116
1.16	.5096	.5075	.5055	.5035	.5015	.4995	.4975	.4956	.4936	.4917
1.17	.4898	.4879	.4860	.4842	.4823	.4805	.4787	.4769	.4751	.4733
1.18	.4715	.4698	.4680	.4663	.4646	.4629	.4612	.4595	.4579	.4562
1.19	.4546	.4529	.4513	.4497	.4481	.4465	.4449	.4434	.4418	.4403
1.20	0.4387	0.4372	0.4357	0.4342	0.4327	0.4312	0.4297	0.4283	0.4268	0.4254
1.21	.4240	.4225	.4211	.4197	.4183	.4169	.4155	.4142	.4128	.4114
1.22	.4101	.4088	.4074	.4061	.4048	.4035	.4022	.4009	.3996	.3983
1.23	.3971	.3958	.3946	.3933	.3921	.3909	.3896	.3884	.3872	.3860
1.24	.3848	.3836	.3825	.3813	.3801	.3790	.3778	.3767	.3755	.3744
1.25	.3733	.3721	.3710	.3699	.3688	.3677	.3666	.3656	.3645	.3634
1.26	.3623	.3613	.3602	.3592	.3581	.3571	.3561	.3550	.3540	.3530
1.27	.3520	.3510	.3500	.3490	.3480	.3470	.3460	.3451	.3441	.3431
1.28	.3422	.3412	.3403	.3393	.3384	.3374	.3365	.3356	.3347	.3338
1.29	.3328	.3319	.3310	.3301	.3392	.3284	.3275	.3266	.3257	.3248
1.3	0.3240	0.3155	0.3075	0.2998	0.2924	0.2854	0.2876	0.2721	0.2659	0.2600
1.4	.2542	.2487	.2434	.2383	.2333	.2286	.2240	.2195	.2152	.2111
1.5	.2071	.2032	.1994	.1958	.1922	.1888	.1855	.1822	.1791	.1760
1.6	.1731	.1702	.1674	.1647	.1620	.1594	.1569	.1544	.1520	.1497
1.7	.1474	.1452	.1431	.1409	.1389	.1369	.1349	.1330	.1311	.1293
1.8	.1275	.1257	.1240	.1223	.1207	.1191	.1175	.1160	.1145	.1130
1.9	.1116	.1102	.1088	.1074	.1061	.1048	.1035	.1022	.1010	.0998
2.0	0.0986	0.0974	0.0963	0.0952	0.0941	0.0930	0.0920	0.0909	0.0899	0.0889
2.1	.0879	.0869	.0860	.0850	.0841	.0832	.0823	.0814	.0806	.0797
2.2	.0789	.0781	.0773	.0765	.0757	.0750	.0742	.0735	.0727	.0720
2.3	.0713	.0706	.0699	.0692	.0686	.0679	.0673	.0666	.0660	.0654
2.4	.0648	.0642	.0636	.0630	.0624	.0618	.0613	.0607	.0602	.0596
2.5	.0591	.0586	.0581	.0576	.0571	.0566	.0561	.0556	.0551	.0547
2.6	.0542	.0538	.0533	.0529	.0524	.0520	.0516	.0511	.0507	.0503
2.7	.0499	.0495	.0491	.0487	.0483	.0479	.0476	.0472	.0468	.0465
2.8	.0461	.0457	.0454	.0450	.0447	.0444	.0440	.0437	.0434	.0430
2.9	.0427	.0424	.0421	.0418	.0415	.0412	.0409	.0406	.0403	.0400
3.0	0.0397	0.0394	0.0392	0.0389	0.0386	0.0383	0.0381	0.0378	0.0375	0.0373
3.1	.0370	.0368	.0365	.0363	.0360	.0358	.0355	.0353	.0351	.0348
3.0	0.0397	0.0370	0.0346	0.0324	0.0304	0.0286	0.0270	0.0255	0.0241	0.0228
4.0	.0217	.0206	.0196	.0186	.0178	.0170	.0162	.0155	.0148	.0142
5.0	.0137	.0131	.0126	.0121	.0117	.0113	.0108	.0104	.0101	.0098
	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	
	0.0094	0.0080	0.0069	0.0060	0.0052	0.0047	0.0042	0.0037	0.0034	

$$Q_1'(z) = \frac{1}{2}\log_e \frac{z+1}{z-1} - \frac{z}{z^2-1}$$

Below the line * * * *, second differences are negligible. Prefix the sign − to every value

z	0	1	2	3	4	5	6	7	8	9
1.10	3.7158	3.6710	3.6270	3.5839	3.5417	3.5004	3.4598	3.4201	3.3811	3.3429
1.11	3.3054	3.2687	3.2326	3.1972	3.1624	3.1283	3.0948	3.0619	3.0296	2.9979
1.12	2.9667	2.9360	2.9059	2.8764	2.8473	2.8187	2.7906	2.7629	2.7358	2.7090
	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *
1.13	2.6827	2.6569	2.6314	2.6063	2.5817	2.5574	2.5335	2.5100	2.4869	2.4641
1.14	.4416	.4195	.3977	.3763	.3551	.3343	.3138	.2935	.2736	.2540
1.15	.2346	.2155	.1967	.1781	.1598	.1418	.1240	.1064	.0891	.0720
1.16	.0551	.0385	.0221	.0059	1.9899	1.9741	1.9586	1.9432	1.9280	1.9130
1.17	1.8982	1.8836	1.8692	1.8550	.8409	.8270	.8133	.7998	.7864	.7731
1.18	.7601	.7472	.7344	.7218	.7093	.6970	.6849	.6728	.6609	.6492
1.19	.6376	.6261	.6147	.6035	.5924	.5814	.5706	.5598	.5492	.5387
1.20	1.5283	1.5180	1.5079	1.4978	1.4879	1.4870	1.4683	1.4587	1.4491	1.4397
1.21	.4304	.4211	.4120	.4030	.3940	.3851	.3764	.3677	.3589	.3506
1.22	.3421	.3338	.3255	.3174	.3093	.3012	.2933	.2854	.2776	.2699
1.23	.2623	.2547	.2472	.2398	.2325	.2252	.2180	.2108	.2037	.1967
1.24	.1898	.1829	.1760	.1693	.1626	.1559	.1494	.1428	.1364	.1300
1.25	.1236	.1173	.1111	.1049	.0988	.0927	.0867	.0807	.0748	.0689
1.26	.0631	.0573	.0516	.0459	.0403	.0347	.0292	.0237	.0183	.0129
1.27	.0076	.0023	0.9970	0.9918	0.9866	0.9815	0.9764	0.9713	0.9663	0.9614
1.28	0.9564	0.9516	.9467	.9419	.9371	.9324	.9377	.9230	.9184	.9138
1.29	.9093	.9047	.9003	.8958	.8914	.8870	.8827	.8784	.8741	.8698
1.30	0.8656	0.8614	0.8573	0.8532	0.8491	0.8450	0.8410	0.8370	0.8330	0.8291
1.31	.8251	.8212	.8174	.8136	.8098	.8060	.8022	.7985	.7948	.7912
1.32	.7875	.7839	.7803	.7767	.7732	.7697	.7662	.7627	.7593	.7559
1.33	.7525	.7491	.7458	.7424	.7391	.7359	.7326	.7294	.7261	.7230
1.34	.7198	.7166	.7135	.7104	.7073	.7042	.7012	.6982	.6952	.6922
1.35	.6892	.6863	.6833	.6804	.6775	.6747	.6718	.6690	.6662	.6634
1.36	.6606	.6578	.6551	.6524	.6497	.6470	.6443	.6416	.6390	.6364
1.37	.6338	.6312	.6286	.6260	.6235	.6209	.6184	.6159	.6134	.6110
1.38	.6085	.6061	.6037	.6013	.5989	.5965	.5941	.5918	.5894	.5871
1.39	.5848	.5825	.5802	.5780	.5757	.5735	.5712	.5690	.5668	.5646
1.40	0.5625	0.5603	0.5581	0.5560	0.5539	0.5518	0.5497	0.5476	0.5455	0.5434
1.41	.5414	.5393	.5373	.5353	.5333	.5313	.5293	.5273	.5253	.5234
1.42	.5215	.5195	.5176	.5157	.5138	.5119	.5100	.5082	.5063	.5045
1.43	.5026	.5008	.4990	.4972	.4954	.4936	.4918	.4900	.4883	.4865
1.44	.4848	.4831	.4813	.4796	.4779	.4762	.4745	.4729	.4712	.4695
1.45	.4679	.4663	.4646	.4630	.4614	.4598	.4582	.4566	.4550	.4534
1.46	.4519	.4503	.4488	.4472	.4457	.4442	.4426	.4411	.4396	.4381
1.47	.4366	.4352	.4337	.4322	.4308	.4293	.4279	.4264	.4250	.4236
1.48	.4222	.4208	.4194	.4180	.4166	.4152	.4138	.4125	.4111	.4097
1.49	.4084	.4071	.4057	.4044	.4031	.4018	.4005	.3991	.3979	.3966

x	0	1	2	3	4	5	6	7	8	9
1.50	0.3953	0.3940	0.3927	0.3915	0.3902	0.3890	0.3877	0.3865	0.3852	0.3840
1.51	.3828	.3816	.3804	.3791	.3779	.3768	.3756	.3744	.3732	.3720
1.52	.3709	.3697	.3685	.3674	.3662	.3651	.3640	.3628	.3617	.3606
1.53	.3595	.3584	.3573	.3562	.3551	.3540	.3529	.3518	.3507	.3497
1.54	.3486	.3475	.3465	.3454	.3444	.3433	.3423	.3413	.3402	.3392
1.55	.3382	.3372	.3362	.3352	.3342	.3332	.3322	.3312	.3302	.3292
1.56	.3283	.3273	.3263	.3254	.3244	.3234	.3225	.3215	.3206	.3197
1.57	.3187	.3178	.3169	.3160	.3150	.3141	.3132	.3123	.3114	.3105
1.58	.3096	.3087	.3078	.3069	.3061	.3052	.3043	.3034	.3026	.3017
1.59	.3009	.3000	.2992	.2983	.2975	.2966	.2958	.2950	.2941	.2933
1.60	0.2925	0.2917	0.2908	0.2900	0.2892	0.2884	0.2876	0.2868	0.2860	0.2852
1.61	.2844	.2836	.2828	.2821	.2813	.2805	.2797	.2790	.2782	.2774
1.6	0.2925	0.2844	0.2767	0.2693	0.2621	0.2552	0.2486	0.2423	0.2361	0.2302
1.7	.2245	.2190	.2137	.2086	.2036	.1988	.1942	.1898	.1854	.1812
1.8	.1772	.1733	.1695	.1658	.1622	.1588	.1554	.1521	.1490	.1459
1.9	.1429	.1400	.1372	.1345	.1318	.1293	.1267	.1243	.1219	.1196
2.0	.1174	.1152	.1130	.1110	.1089	.1069	.1050	.1031	.1013	.0995
2.1	.0978	.0961	.0944	.0928	.0913	.0897	.0882	.0867	.0853	.0839
2.2	.0825	.0812	.0799	.0786	.0773	.0761	.0749	.0737	.0726	.0714
2.3	.0704	.0693	.0682	.0672	.0662	.0652	.0642	.0633	.0624	.0614
2.4	.0605	.0597	.0588	.0580	.0572	.0563	.0556	.0548	.0540	.0533
2.5	.0525	.0518	.0511	.0504	.0497	.0491	.0484	.0478	.0472	.0465
2.6	.0459	.0453	.0447	.0442	.0436	.0430	.0425	.0420	.0414	.0409
2.7	.0404	.0399	.0394	.0389	.0384	.0380	.0375	.0371	.0366	.0362
2.8	.0357	.0353	.0349	.0345	.0341	.0337	.0333	.0329	.0325	.0322
2.9	.0318	.0314	.0311	.0307	.0304	.0300	.0297	.0294	.0291	.0287
3.0	.0284	.0281	.0278	.0275	.0272	.0269	.0266	.0263	.0261	.0258
3.1	0.0255	0.0253	0.0250	0.0247	0.0245	0.0242	0.0240	0.0237	0.0235	0.0232
3.0	0.0284	0.0255	0.0230	0.0208	0.0189	0.0172	0.0157	0.0144	0.0132	0.0122
4.0	.0113	.0104	.0096	.0090	.0083	.0078	.0073	.0068	.0064	.0060
5.0	.0056	.0053	.0050	.0047	.0044	.0042	.0039	.0037	.0035	.0034
	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	
	0.0032	0.0025	0.0020	0.0016	0.0013	0.0011	0.0009	0.0008	0.0007	

The above tables were computed to five places with Barlow's Tables of squares and Naperian logarithms, checked by differences and the last figure cut off. When the figure cut off was exactly 5, the number was recomputed to six places or seven places.

ON THE MEAN AGE AT DEATH OF CENTENARIANS*

By Raymond Pearl

School of Hygiene and Public Health, Johns Hopkins University

Communicated, January 28, 1919

Some years ago I investigated the mean age at death of infants dying at very early ages.[1] The purpose of that study was to determine with accuracy where the deaths under one year should be centered in statistical computations involving the computation of moments, where one had to know the center

TABLE 1

Age Distribution of Centenarian Deaths in the Registration Area, 1916

AGE.	MALES		FEMALES	
	White	Colored	White	Colored
years				
100	48	42	72	85
101	15	4	30	8
102	14	10	25	19
103	12	11	15	14
104	17	5	9	14
105	14	8	18	27
106	2	7	5	8
107	5	1	1	2
108	2	6	1	5
109	1	1	0	2
110	4	7	2	15
111	1	4	0	3
112	0	1	0	3
113	0	2	0	0
114	2	0	1	1
115	0	2	1	4
116	0	1	0	3
118	0	2	0	0
120	0	2	0	2
134	0	0	0	1
Totals.....	137	116	180	216

of gravity of each frequency area over given base units. In an investigation which I am now undertaking I have been confronted with the same problem at the other end of the life span. In the "Mortality Statistics" published by the Bureau of the Census, the deaths occurring at ages of 100 and over are lumped in a single class. In calculating the frequency constants of mor-

* Papers from the Department of Biometry and Vital Statistics, School of Hygiene and Public Health, Johns Hopkins University, No. 3.

tality curves, it is obvious that these centenarian deaths must be centered at some point. The finding of as close an approximation as possible to the correct centering point has been attacked along two lines, with the results set forth in this paper.

In the first place, actual deaths occurring in the year 1916 in the Registration Area of individuals reported to the Census Bureau as having died at age 100 or more were studied. Detailed statistics giving the distribution of these deaths by years was very kindly furnished me by Dr. William H. Davis, Chief Statistician of the Vital Statistics Division, Bureau of the Census. I wish to record here my obligation to Doctor Davis for furnishing me with this tabulation.

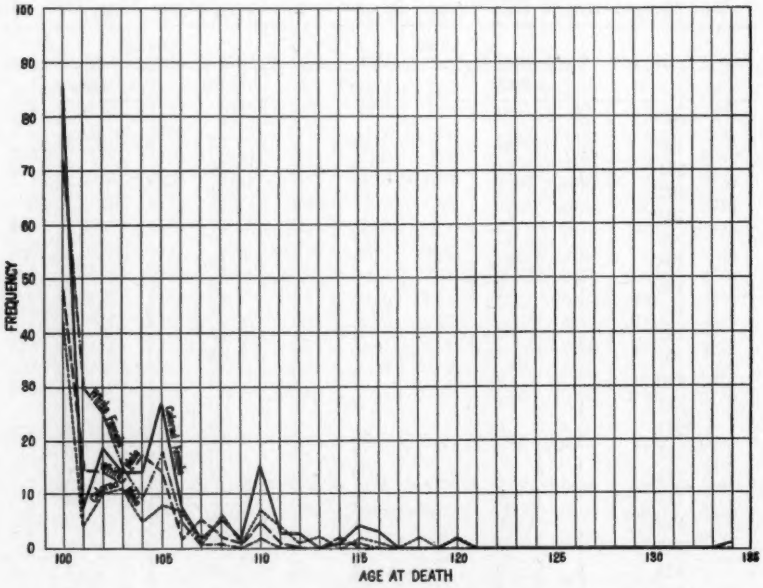


FIG. 1. DISTRIBUTION OF THE DEPTHS IN THE REGISTRATION AREA IN 1916, AT AGE 100 OR MORE

These figures are set forth in table 1, and graphically in figure 1.

It is apparent at once from the table and diagram that there are gross errors in the records of ages in these centenarian deaths. Unquestionably, some people do live to very advanced ages[2] but it is equally clear that they do not occur in such numbers, nor at such advanced ages as would be indicated by these figures, taken at their face values.

Particularly are the figures for the colored population obviously incorrect. One notes the outstanding humps in the colored female line at the quinquennial years, 100, 105, 110, etc. These, of course, have no foundation in fact. In

the case of the whites, the statistics are undoubtedly more accurate, and one would certainly not be justfied in going so far as does Clark[3] in denying the existence of centenarians except in the very rarest cases.

From the unmodified data in table 1, the constants shown in table 2 have been deduced.

Before commenting on the data of table 2, it is desirable to present the results of another method of approach to the matter. That method is to take the number of deaths for each year over 100 from the d_x column of a properly graduated mortality table, and calculate from such figures the means and standard deviations. The mortality tables which have been used for this

TABLE 2

CONSTANTS FOR DEATHS OF CENTENARIANS IN THE REGISTRATION AREA IN 1916

GROUP	MEAN	STANDARD DEVIATION
	years	*years*
White males	103.19 ± 0.17	3.02 ± 0.12
White females	102.42 ± 0.12	2.49 ± 0.09
White, both sexes together	102.75 ± 0.10	2.76 ± 0.07
Colored, both sexes together	104.45 ± 0.18	4.89 ± 0.13

TABLE 3

DEATHS OF CENTENARIANS FROM UNITED STATES LIFE TABLES

x TO $x+1$	d_x OF 100,000 BORN ALIVE			
	White males	White females	Negro males	Negro females
100–101	13	19	13	34
101–102	8	11	10	26
102–103	5	6	7	18
103–104	3	4	4	13
104–105	1	2	3	9
105–106	1	1	1	5
106–107	0	1	1	3
107–108	0	0	1	2
108–109	0	0	0	1
109–110	0	0	0	1

purpose are Glover's[4] 1910 American mortality tables. The distributions of death which these tables give for white and colored are exhibited in table 3.

As would be expected the distributions in table 3 are much smoother than those of table 1. Furthermore, their range is less extended at the upper end. The constants for the distributions in table 3 are exhibited in table 4.

From these tables the following points are to be noted:

1. As was to be expected the mean ages at death are sensibly lower, and the standard deviations smaller, when the graduated data of the life tables are used instead of the raw statistics.

2. Inasmuch as the life table values given in table 4 are certainly nearer the true values than those of table 2, they may well be adopted as the basis of centering deaths over 100 in biostatistical computations.

3. Accordingly in this laboratory the deaths of whites, both male and female, occurring at 100 years and over will be assumed to center at 101.7

TABLE 4

MEAN AGE AT DEATH AND STANDARD DEVIATION IN AGE AT DEATH OF CENTENARIANS, FROM UNITED STATES LIFE TABLE FIGURES

GROUP	MEAN	STANDARD DEVIATION
White males	101.66	1.32
White females	101.73	1.49
Whites, both sexes	101.70	1.42
Negroes, both sexes	102.32	1.92

years, and those of negroes at 102.0. The latter value is probably still too high, but unless one repudiates the statistical data entirely, which would amount to throwing away the child with the bath, there is no warrant for arbitrarily taking a lower value.

[1] Pearl, R., *Biometrika*, 4, 1906, (510–516).

[2] Cf. the authentic case reported by Rodriquez (in *Siglo med., Madrid*, 53, 1906, (341–343) > of death at 124 years of age.

[3] Clark, F. C., *Providence, Med. J.*, 10, 1909, (143–158).

[4] Glover, J. W., United States Life Tables, 1910. Bureau of the Census 1916.

REPORT OF THE AUTUMN MEETING

PREPARED BY THE ACTING HOME SECRETARY

The Autumn Meeting of the Academy was held at the Johns Hopkins University in Baltimore, November 18 and 19, 1918.

Forty-one members were present as follows: C. G. Abbot, Abel, Ames, Bliss, D. H. Campbell, Castle, F. W. Clarke, J. M. Clarke, Conklin, Cross, Davenport, Davis, Franklin, Freeman, Gomberg, Halsted, Holmes, Howard, Howe, Howell, Iddings, Jennings, Jones, Leuschner, Lewis, Lindgren, C. E. Mendenhall, J. C. Merriam, Michelson, Moulton, E. F. Nichols, Pupin, Ransome, Russell, Erwin F. Smith, Thorndyke, Vaughan, Walcott, Webster, Welch, R. W. Wood.

BUSINESS SESSIONS

The President announced the death of S. W. Williston and of G. K. Gilbert.

The following assignments of Biographical Memoirs was announced: Memoirs of G. K. Gilbert to William M. Davis, of Richmond Mayo Smith to Edwin R. A. Sigman, and of Samuel W. Williston to Richard S. Lull.

Announcement was made of the following funds available for the National Research Council:

Carnegie Institution	$100,000.00
W. W. Keen	483.57
Rockefeller Foundation	50,000.00
President's Fund	120,000.00
Total available for the fiscal year 1919	$270,483.57

The President announced the following appointments to the Board of Editors of the PROCEEDINGS with terms expiring December, 1921: J. M. Clarke, Ludvig Hektoen, H. S. Jennings, R. A. Millikan, W. A. Noyes.

The Managing Editor of the PROCEEDINGS, Mr. E. B. Wilson, was re-elected for the year ending November 30, 1919.

The following communication from the Home Secretary, Mr. Arthur L. Day, was read and acted upon:

Dr. Charles D. Walcott
President, National Academy of Sciences,
Washington, D. C.

9 Sept. 1918

Dear Dr. Walcott:

Twice within the year, under the pressure of war activities which have kept me away from Washington considerably more than half the time, I have asked to be relieved of the duties of Home Secretary of the Academy. Both times the Council has done me the honor to request that the resignation be withdrawn, even though the duties which I have hitherto performed are required to be otherwise provided for. I have appreciated this very unusual

mark of confidence most keenly, and notwithstanding some twinges of a New England conscience about receiving credit for work done by some one else, I have been very grateful to the Council for its complimentary action.

More recently, it may have come to your ears that I have undertaken a very much greater war responsibility than any with which I have heretofore had to do. It is in connection with the Corning Glass Works at Corning, N. Y., and I have accordingly decided to move my family there.

It is a very necessary though unwritten law that the Home Secretary should be a resident of Washington, which after October 1 I shall no longer be. I therefore respectfully offer a final request to be relieved of the duties of Home Secretary from October 1, 1918.

With kindest regards, believe me,

Very sincerely yours,
ARTHUR L. DAY, *Home Secretary.*

The recommendation of the Council that the resignation of Mr. A. L. Day as Home Secretary be accepted with an expression of appreciation of his valuable services in that office and that the election of a Home Secretary as provided for under the Constitution be held at the next stated meeting of the Academy was approved.

The President presented the following communication from the President of the United States relative to the *National Research Council:*

Executive Order Issued by the President of the United States, May 11, 1918

The National Research Council was organized in 1916 at the request of the President of the National Academy of Sciences, under its Congressional charter, as a measure of national preparedness. The work accomplished by the Council in organizing research and in securing coöperation of military and civilian agencies in the solution of military problems demonstrates its capacity for larger service. The National Academy of Sciences is therefore requested to perpetuate the National Research Council, the duties of which shall be as follows:

1. In general, to stimulate research in the mathematical, physical and biological sciences, and in the application of these sciences to engineering, agriculture, medicine and other useful arts, with the object of increasing knowledge, of strengthening the national defense, and of contributing in other ways to the public welfare.

2. To survey the larger possibilities of science, to formulate comprehensive projects of research, and to develop effective means of utilizing the scientific and technical resources of the country for dealing with these objects.

3. To promote coöperation in research, at home and abroad, in order to secure concentration of effort, minimize duplication, and stimulate progress; but in all coöperative undertakings to give encouragement to individual initiative, as fundamentally important to the advancement of science.

4. To serve as a means of bringing American and foreign investigators into active cooperation with the scientific and technical services of the War and Navy Departments and with those of the civil branches of the government.

5. To direct the attention of scientific and technical investigators to the present importance of military and industrial problems in connection with the war, and to aid in the solution of these problems by organizing specific researches.

6. To gather and collate scientific and technical information at home and abroad, in coöperation with Governmental and other agencies and to render such information available to duly accredited persons.

Effective prosecution of the Council's work requires the cordial collaboration of the scientific and technical branches of the Government, both military and civil. To this end

representatives of the Government, upon the nomination of the President of the National Academy of Sciences, will be designated by the President as members of the Council, as heretofore, and the heads of the departments immediately concerned will continue to cooperate in every way that may be required.

WOODROW WILSON.

THE WHITE HOUSE, *May 11, 1918.*

The President read the following resolutions adopted at the Inter-Allied Conference, held at London, October 9, 1918, received from the Foreign Secretary, Mr. Hale, who was also Chairman of the Delegation, the other members being Messrs. A. A. Noyes, Durand, Flexner, Bumstead, and Carty.

RESOLVED THAT: A Committee of Enquiry be constituted by the Conference, the Academies of the countries at war with the Central Powers having power to add further members. This Committee shall prepare a general scheme of international organizations to meet the requirements of the various branches of scientific and industrial research including those relating to national defence. (The Committee will meet in Paris during the second fortnight in November.)

Each of the Academies represented at the Conference shall be invited to initiate the formation of a National Council for the promotion of the researches specified in Resolution 4.

An International Council, having as nucleus the Committee specified in Resolution 4, shall be formed by the federation of the National Councils.

The Conference being of opinion that all industrial, agricultural and medical progress depends on pure science, draws the attention of the various governments to the importance of theoretical and disinterested researches, which after the restoration of peace should be supported by large endowments. The Conference urges similarly the creation of large laboratories for experimental science, both private and national.

The action of the Academy delegates in having the above resolutions adopted by the Inter-Allied Conference in London was approved by the Academy.

On the recommendation of the Council the Academy placed itself on record as welcoming the publications of a scientific and technical character to go into the building that is contemplated.

Mr. Davis presented the following forms, asking that the members of the Academy sign the petition. Many members present did so, and after the meeting, with the approval of the President of the Academy, additional signatures of members were obtained by correspondence to a total number of about 100.

The undersigned Members of the National Academy of Sciences, meeting in Baltimore November 18, 1918, petition the Congress of the United States to take action in consultation with the governments of many other countries, toward the formation at as early a date as possible of a League of Nations for the maintenance of Peace.

The undersigned Members of the National Academy of Sciences, meeting in Baltimore November 18, 1918, having petitioned the Congress of the United States to take action, in consultation with the governments of many other countries, toward the formation at as early a date as possible of a League for the *maintenance* of Peace, hereby urge the members of other learned societies in the United States to do likewise.

The following motion was offered:

MOVED: That the Home Secretary be requested to transmit the thanks of the Academy to the President and Trustees of Johns Hopkins University, the members of the Johns Hopkins Club and the members of the Academy resident in Baltimore serving as the local committee, for the courtesies extended to the members of the National Academy of Sciences at the Autumn Meeting, 1918. (*Adopted.*)

SCIENTIFIC SESSIONS

Three public scientific sessions were held on November 18 and 19 at which the following papers were presented.

C. G. ABBOT: Cloud reflection and the albedo of the earth and Venus.

A. H. PFUND (introduced by J. S. AMES): Colorimetry of white surfaces.

F. W. CLARKE and G. STEIGER: The inorganic constituents of lobster shells.

R. W. HEGNER (introduced by H. S. JENNINGS): Quantitative relations between chromatin and cytoplasm in the genus Arcella, with their relations to external characters.

D. R. HOOKER (introduced by W. H. HOWELL): The physiological effects of air-concussion.

W. H. HOWELL: Two new factors in blood coagulation.

C. B. DAVENPORT and ALBERT G. LOVE: Comparative morbidity of white and colored troops.

A. G. WEBSTER: Theory of wind instruments.

A. G. WEBSTER: On the ballistic resistance function.

A. G. WEBSTER: On the dynamics of the rifle fired from the shoulder.

G. P. MERRILL: Second report on the researches on the chemical and mineralogical composition of meteorites.

W. E. DANDY (introduced by W. S. HALSTED): Hydrocephalus: experimental and clinical study. Illustrated.

W. S HALSTED: Clinical and experimental observations in cases of arteriovenous and lymphatico-venous fistulae. Illustrated.

W. E. CASTLE: Is the arrangement of the genes in the chromosome linear? Illustrated.

E. F. SMITH: Cause of Phyllomania in Begonia. Illustrated.

W. G. MACCALLUM (introduced by H. S. JENNINGS): Recent epidemics of pneumonia in army camps. Illustrated.

G. A. BLISS: Differential corrections of ballistics.

S. B. WOLBACH: Some results of studies of Influenza at an army camp.

V. C. VAUGHAN: Communicable diseases in an army camp.

R. W. WOOD: Physiological effects of light of short wave-length.

*G. N. LEWIS: Kinetics and thermodynamics.

*A. O. LEUSCHNER: Perturbations and tables of the minor planets discovered by James C. Watson, Part II.

*C. E. VAN ORSTRAND: Mathematical tables.

*F. SLATE, Jr.: Biography of Eugene Waldemar Hilgard.

*W. W. CAMPBELL: Biographical memoir of George Davidson.

*C. R. CROSS: Biographical memoir of James Mason Crafts.

*T. D. A. COCKERELL: Biographical memoir of Alpheus Spring Packard.

*F. SLATE, Jr.: Biographical memoir of Eugene Waldemar Hilgard.

*The asterisk denotes presentation by title.

NATIONAL RESEARCH COUNCIL

MINUTES OF THE MEETING OF THE EXECUTIVE BOARD

AT THE NATIONAL RESEARCH COUNCIL BUILDING, DECEMBER 21, 1918, AT 10 A.M.

Present: Messrs. Bogert, Cross, Flinn, Howe, Hussey, Johnston, Manning, Mendenhall, Merriam, Millikan, Pupin, Walcott, Washburn, Welch, Woods, Woodward. and Yerkes.

The Chairman presented a report of the Committee on Reconstruction Problems. This Committee at its first meeting on August 18, 1918, adopted a program calling for research into the agencies and activities dealing with after-war problems. It appears that many organizations have taken up, or are about to take up, some phase of this far-reaching matter; so that, in theory at least, there are relatively few points not being covered. The Committee decided therefore to limit its efforts to a specific undertaking; and recommends the preparation of a comprehensive report on the whole question of the supply and control of water in relation to food and power production and other industries, a question which enters largely into all reconstruction plans throughout the world. In the compilation of data and preparation of this report it is proposed that each member of the Committee should cover one of the major topics, and that experts in Government service be invited to cooperate; and that the whole should be edited by some one person, with due credit to the several collaborators and sources of information.

Moved: That the plan presented by the Committee on Reconstruction Problems be approved, provided that it does not conflict with similar activities of present Government organizations and that the manuscript of the report having to do with reconstruction be submitted to the Committee on Publicity and Publications for approval before it is sent to press. (*Adopted.*)

Mr. Merriam, Chairman of the Committee on Organization, presented a report, in substance as follows:

The period of the war emergency having come to an end, so far as activities in most phases of research are concerned, the National Research Council must take up for immediate consideration the extent of readjustment necessary in order to adapt its machinery most perfectly for functioning through the period of reconstruction and in normal times of peace.

In the evolution of the Research Council the first stage was that of a temporary organization intended to continue for a period not longer than one year, within which time it was assumed that the best mode of operation would become evident. The entrance of America into the war made necessary considerable modification of the temporary machinery in order to permit concentration of effort within the briefest limits of time and space. These shifts were not presumed to lead in all cases in the direction of permanent organization, and suggestions relating to the ultimate plan upon which the work of the Council was to be based continued under discussion through the period of the war. While it is not necessary to assume that either the aims or the plan of operation of the Council must be materi-

ally altered for the immediate future, it is certain that the tremendous change of conditions in the country incident to ending of the war makes necessary the immediate consideration of adaptability of the Council's present machine for handling the next problems to come before it. Not only must the question as to our present situation be answered for ourselves, but we must make clear to the research men of the country that the Council has fully considered this matter and that, recognizing changing conditions, it is fitting itself as rapidly as possible into the best position for future work.

The aims and purposes of the Research Council have from the beginning covered the broadest relations and applications of science. In recent months the work has been expressed in effort to assist in bringing science to bear upon problems of war. From this experience it has been clear that efficiency of the nation in war is in considerable measure dependent upon the most intimate correlation of scientific interests, such as is also needed in time of peace. With the cessation of hostilities, we find our problem differing largely in mode of application of results of research and in the larger freedom to consider stimulation and correlation of scientific investigation and initiation of new enterprises.

The skeleton of the present war organization of the Research Council appears in general to furnish a satisfactory basis for operation in time of peace. The principal modifications needed involve:

(1) The placing of larger emphasis upon the initiation, stimulation, and correlation of fundamental researches.

(2) Reorganization of the Divisions so as to secure a wide representation of the major research interests in the country regardless of geographic location of the members.

Owing to the immediate need for securing assistance of science in handling emergency problems, the work of the Council has recently tended toward the extreme of application of results of research, rather than toward the stimulation or initiation of work on new problems. With relief from urgent requirements for application of available research data in war, it becomes possible for the investigator to concern himself more particularly with researches in science and technology, depending upon his cooperation with the engineer for application of results secured. At the same time that we are relieved from extraordinary requirements in application of research we are given the possibility of bringing into conference with the Research Council a considerable group of men whose participation in the work of this organization has been difficult because of geographic situation.

Accordingly it is considered that the organization of the National Research Council should be such as to render it an effective federation of the research agencies of the country. To this end a large proportion of the membership of the Council will be nominated by the national scientific and technical societies. All members will be appointed for periods of three years, and will be grouped into a series of divisions each of which will deal with related branches of science and technology or with some specific phase of activity of the Council. Each Division will elect a Chairman, Vice-Chairman and Executive Committee to administer the affairs of the Division. The Chairman and Vice-Chairman of each of the several Divisions, together with certain other members, shall be the Executive Board of the Council; the officers of the Executive Board will be the officers of the Council as a whole. The Executive Board will determine the general policy of the Council; but each Division will be as autonomous as possible, and the policies of the several Divisions may differ in various respects provided that these do not conflict with the general policies of the Council.

Tentative plans, in accordance with the above general principles, were discussed at length; but no final conclusions were reached.

Moved: That the report of the Committee on Organization be approved with the recommendations contained therein. (*Adopted.*)

Moved: That the question of reorganization of the Divisions concerned with administrative, governmental and general relations, be referred to the Committee on Organization

for further consideration and that the Organization Committee in conference with the Divisions take up the problem of modification of the division organization. (*Adopted.*)

Moved: That the Executive Board ask for a conference at an early date with the Council of the National Academy of Sciences for the consideration of the recommendations set forth in the plans presented by the Committee on Organization. (*Adopted.*)

By request of Mr. Phinehas Stephens attention of the Research Council was called to the Smith-Howard Bill, the general purpose of which is to give grants of federal money to the several states for engineering experiment stations.

Moved: That a committee of six be appointed by the Chairman to recommend what action, if any, shall be taken by the National Research Council. (*Adopted.*)

The Chairman appointed as members of the Committee Messrs. Carty, Dunn (Chairman), Howe, Leuschner, Millikan and Pupin.

In response to a notice of the Secretary of the National Research Council dated November 29, a meeting of the Special Committee appointed to discuss the proposals in the Smith-Howard Bill for the establishment of engineering experiment stations was held at the offices of the National Research Council in Washington on Wednesday, December 4, 1918, at 9.30 a.m. There were present Messrs. Henry M. Howe, F. B. Jewett, John Johnston, A. O. Leuschner, J. C. Merriam, R. A. Millikan, M. I. Pupin, S. W. Stratton, C. D. Walcott, W. R. Whitney, and Gano Dunn, Chairman. Messrs. R. C. Maclaurin and Van H. Manning were unavoidably absent and Mr. Manning was represented by Mr. H. Foster Bain. By invitation Mr. Phinehas V. Stephens, author of the Smith-Howard Bill, attended the latter part of the meeting and participated in the discussion.

The text of the Smith-Howard Bill, as of November 8, 1918, was thoroughly analyzed and discussed and the text of a proposed substitute drafted by Major Leuschner was offered and considered. After much discussion certain principles were adopted as expressing the judgment of the Special Committee.

The report of the Special Committee was presented to the Executive Board and approved by it; and a small committee consisting of Messrs. Leuschner (Chairman), Howe, Johnston, and Pupin, was appointed to incorporate in a revised text of the Smith-Howard Bill the principles laid down by the Special Committee.

A revised text of the Smith-Howard Bill, presented to the Executive Board, was agreed to by the Board.

Mr. Howe reported that the Committee on Helmets and Body Armor had subscribed among themselves nearly $2500 for carrying on the work of that Committee.

Moved: That a vote of thanks of the National Research Council be given to the members of the Committee on Helmets and Body Armor for their interest and support in carrying forward the research work of that Committee. (*Adopted.*)

Moved: That Bradley Stoughton be elected as a Second Vice-Chairman of the Division of Engineering. (*Adopted.*)

Moved: That Armin O. Leuschner be elected to represent astronomy on the Executive Committee of the Division of Physics, Mathematics, Astronomy, and Geophysics. (*Adopted.*)

Moved: That the resignation of J. C. Merriam as Chairman of the Division of Geology and Geography be accepted with great regret; and that the National Research Council express its appreciation of the great services which he has rendered to the Council and convey to him a vote of thanks. (*Adopted.*)

Moved: That Mr. Whitman Cross be elected Chairman of the Division of Geology and Geography, and that Mr. P. S. Smith be elected Vice-Chairman. (*Adopted.*)

Mr. Rous stated that on December 4 the Rockefeller Foundation adopted the following resolution:

RESOLVED that the sum of Fifteen Thousand Dollars ($15,000) be, and it is hereby appropriated, of which so much as may be necessary shall be paid to the NATIONAL RESEARCH COUNCIL for the special work in connection with the war emergency and the demobilization period, of the *Division of Medicine and Related Sciences* for the year 1919.

Moved: That the National Research Council transmit to the Rockefeller Foundation a vote of appreciation and thanks for its splendid support of the Division of Medicine and Related Sciences in its work carried on during the war emergency and the period of demobilization. (*Adopted.*)

Moved: That the resignation, as of December 31, 1918, of R. M. Pearce and of Peyton Rous be accepted with regret; and that the National Research Council express its appreciation of the splendid work which they have accomplished as heads of the Division of Medicine and Related Sciences, and convey to them a vote of thanks for their assistance. (*Adopted.*)

Moved: That the election of Colonel Russell as Acting Chairman and that of Major Hussey as Vice-Chairman of the Division of Medicine and Related Sciences be approved. (*Adopted.*)

The meeting adjourned at 12.50.

PAUL BROCKETT, *Assistant Secretary.*